TOTA 2021

Trends in Operator Theory and its Applications

BOOK OF ABSTRACTS

Department of Mathematics
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Contents

Ergodicity for Markov semigroups and applications to singular SDEs

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Abstract

The aim of this talk is to present some classic and recent results concerning existence and uniqueness of invariant distributions for Markov semigroups (or stochastic processes), as well as their speed of convergence to equilibrium. If time allows, we shall discuss some applications to singular stochastic differential equations in infinite dimensions.

Keywords: Markov semigroups, ergodicity, singular SDEs.

Jordan structures living inside operator spaces

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Abstract

The lack of associativity frequently means an obstacle when we deal with Jordan structures. However, under the right conditions, it is sometimes possible to define favourable frameworks to work in. This talk is intended to briefly review some known results which allow us to regard certain Jordan algebras as operator spaces. We shall also state our own contribution to the topic, included in a joint work with A.M. Peralta.
Keywords: JB*-algebras, JC*-algebras, Shirshov-Cohn type theorem.

References

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Interpolating Matrices

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Abstract

We extend Carleson's interpolation Theorem to sequences of square matrices with eigenvalues in the unit disc, without making any assumptions on the sequence of their sizes. This will be done by adapting the operator theoretical approach to interpolation to this matrix-nodes problem. As for the scalar case, separation conditions with with respect bounded analytic functions correspond to separated model sub-spaces of the Hardy space. Such model spaces work as reproducing kernels at a matrix.

Keywords: Interpolating Sequences, Separated Model Spaces

Amplitude Equations in Fluid Mechanics

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Abstract

Global modes and the amplitude equations that describe have been the focus of many authors in Fluid Mechanics. The failure of global modes to describe fluid flows has been related to the non-normality and nonlinearity in the governing equations (Chomaz 2005). We demonstrate that for strongly non-normal linear operator, it is impossible to capture the saturation characteristics of flow with the global mode governed for the simple reason that, as the flow becomes more non-normal, the projection of the flow onto the global mode becomes less and less of the entire flow. We propose a remedy to this via approximating the solution via a non-self-adjoint perturbation expansion (Vishik and Lyusternik 1960). The test example that we choose is the non-self-adjoint Ginzburg-Landau equation. We consider this operator with real coefficients and complex coefficients. In the former case, an, albeit unbounded, metric operator generating an underlying lattice of Hilbert Spaces. A Riesz-basis-system therefore exists, which allows us to obtain higher order coefficients correctly with two possible normalisation choices. In the case of the Complex Ginzburg-Landau equation, this situation does not occur owing to the absence of the metric operator.

Keywords: Amplitude Equations, Ginzburg-Landau, Reduced-Order-Modelling.

Reducing Subspaces For Rank-One Perturbations Of Diagonal Operators

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Abstract

In this talk, we will address the existence of reducing subspaces for operators T which are rank-one perturbations of normal operators, and in particular of diagonal operators, acting on an infinite dimensional, separable, complex Hilbert space H. As we will show, the spectral picture will be essential. In addition, we will characterize the reducing subspaces M such that the restriction $T\mid_M$ is normal linking them to a classical theorem of Behncke about the orthogonal decomposition of H in terms of the reducing subspaces of the operator T. This is a joint work with Eva A. Gallardo-Gutiérrez.

Keywords: reducing subspaces, rank-one perturbation of diagonal operators, rank-one perturbation of normal operators.

Analogues of finite Blaschke products as inner functions

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Abstract

In a fairly general setting, we will discuss analogues of finite Blaschke products as they relate to inner functions. These functions can be seen as certain linear combinations of reproducing kernels, and as generalizations of a construction given by Shapiro and Shields. Joint work with Trieu Le.

Keywords: Blaschke products, inner functions.

The dynamics of weighted composition operators on Fock spaces

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Abstract

The study of weighted composition operators acting on spaces of analytic functions has recently developed into an active area of research. In particular, characterisations of the bounded and compact weighted composition operators acting on Fock spaces were identified by, amongst others, Ueki [4], Le [2], and Tien and Khoi [3].

In this talk I will examine some recent results [1], that give explicit descriptions of bounded and compact weighted composition operators acting on Fock spaces. This allows us to prove that Fock spaces do not support supercyclic weighted composition operators. This is joint work with Tom Carroll (University College Cork, Ireland).

Keywords: Weighted composition operators, Fock spaces, linear dynamics.

References

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On the solution of a class of integral equations using new weighted convolutions

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Abstract

This talk is devoted to the study of the solvability of a class of integral equations, whose kernel depends on four different functions. To obtain necessary and sufficient conditions for the unique solution of this class of equations, we introduce eight new weighted convolutions somehow associated with the kernel of our equations. Moreover, we obtain two Young-type inequalities associated with each convolution.

Keywords: convolution, weight function, Young's inequality, integral equation, factorization identity.

Adjoints of Composition Operators with Quadratic Rational Symbols

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Abstract

The composition operator C_φ, with symbol φ, is defined by $C_\varphi f = f \circ \varphi$ for f in a Hilbert space of analytic functions on the unit disk. If φ is a self-map of the unit disk that is not univalent, it is well known that the kernel of the adjoint C_φ^* of the composition operator is infinite-dimensional. In this talk, we classify functions in the kernel of C_φ^*, with quadratic rational symbols, in classical Hilbert spaces of analytic functions.

Keywords: composition operator, adjoint, kernel.

Convolution weights on ℓ_2-spaces

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Abstract

It is known that the weighted L_p-space on a locally compact group is stable with respect to convolution if the weight function satisfies a certain type of convolution inequality. There are several counterexamples showing that this sufficient condition is not necessary. However, for one class of groups, namely discrete abelian groups, no such counterexample is known. Thus there remains the possibility that the convolution inequality truly characterizes stability of convolution for weighted L_p-spaces on these groups. In this talk, we investigate this inequality and, in the case $p = 2$, reinterpret it in the light of operator theory and in the context of the theory of reproducing kernel Hilbert spaces.

Keywords: Weighted L_p-space, Locally compact group, Convolution algebra.

Pseudomode for biharmonic operators with complex potentials

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Abstract

This talk is about the construction of pseudomodes of one-dimensional non-self-adjoint biharmonic operators corresponding to large pseudoeigenvalues. This is the first systematic non-semiclassical approach that can cover a wide class of previously inaccessible potentials, including super-exponential ones.

Keywords: Pseudospectrum, WKB, biharmonic operators.

Symbols of truncated Toeplitz operators

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Abstract

When Sarason initiated the study of truncated Toeplitz operators in 2007 he posed the question, does every bounded truncated Toeplitz operator have a bounded symbol? This was then shown to be negative in 2010 when a bounded truncated Toeplitz operator was constructed which does not possess a bounded symbol. Further work then gave a characterisation of when a bounded truncated Toeplitz operator has a bounded symbol. In this talk I will consider analogous questions looking at the role of continuous symbols for compact truncated Toeplitz operators.

Keywords: Toeplitz operator, Hardy space .

On the reachable space of the Hermite heat equation with boundary control

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Abstract

The description of the reachable space of parabolic equations is a central problem in control theory. For the one-dimensional heat equation with boundary control, this problem has been the source of intensive research culminating with an exact characterization both on the half-line $(0,\infty)$ in [2] and on a segment in [1]. We proved in this last paper that the reachable states of the heat equation extend holomorphically and belong to the Bergman space of a square (see the presentation of A. Hartmann).

In this talk, we will focus on the reachable space of another parabolic equation namely the Hermite heat equation, in which the (one-dimensional) Laplacian $-\partial_x^2$ is replaced by the quantum harmonic oscillator $-\partial_x^2 + x^2$. On the first hand, we will see that the reachable on the half-line $(0,\infty)$ is the same as for the heat equation. On the other hand, we will give a sharp result for the equation on a segment showing that the reachable space is very close to the Bergman of a square.

Keywords: Hermite heat equation, reachable space, Bergman space, control theory.

References

[1] A. Hartmann and M-A Orsoni (2021) Separation of singularities for the Bergman space and application to control theory, *Journal de Mathématiques Pures et Appliquées*, to appear.

[2] Saitoh, S. (1991) Isometrical identities and inverse formulas in the one-dimensional heat equation, *Appl. Anal.*, **40** 139–149.

Summability in Banach spaces of holomorphic functions

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Abstract

In this talk, I will present an abstract result in summability theory : if one regular summability method includes another for scalar sequences, then it automatically does so for certain Banach-space-valued sequences too. I will also state some consequences of this result on summability theory in Banach spaces of holomorphic functions on the unit disk.

Keywords: Summability methods, Banach spaces, Banach spaces of holomorphic functions, de Branges-Rovnyak spaces.

A Boundary Value Problem on the Polydisc

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Abstract

In 1955, Lehto showed that, for every measurable function ψ on the unit circle $\mathbb{T}$, there is function f holomorphic in the unit disc $\mathbb{D}$, having ψ as radial limit a.e. on $\boldsymbol{T}$. We consider an analogous boundary value problem in several complex variables.

Keywords: Boundary value problem, Polydisc.

mth roots of H-selfadjoint matrices

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Abstract

A square matrix B is called H-selfadjoint for some invertible Hermitian matrix H if it is self-adjoint in the corresponding indefinite inner product space, or equivalently, if $HB = B^*H$. Let B be an H-selfadjoint complex matrix. The necessary and sufficient conditions are given for the existence of an H-selfadjoint matrix A such that $A^m = B$, that is, A is an mth root of B. It is well known that there exists a complex matrix representation for quaternion matrices and this is used to also look at the quaternion case of the problem.

Keywords: indefinite inner product, H-selfadjoint matrices, roots of matrices.

Invertibility of Fourier convolution operators with piecewise continuous symbols on variable Lebesgue spaces

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Abstract

We extend results on the invertibility of Fourier convolution operators with piecewise continuous symbols on the Lebesgue space $L^p(\mathbb{R})$, $p \in (1, +\infty)$, obtained by Roland Duduchava in the late 1970s, to the setting of variable Lebesgue spaces such that the Hardy-Littlewood maximal operator is bounded on $L^{p(\cdot)}(\mathbb{R})$.

Keywords: Invertibility, Fourier convolution operators, Fourier multiplier, Piecewise continuous functions, Variable Lebesgue spaces.

References

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Numerical index of a Banach space

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Abstract

Numerical index of a Banach space is a number relating the norm and the numerical range of a bounded linear operator. Given a Banach space X, the *numerical radius* of a bounded linear operator T on X is given by

$$v(T) := \sup\{|x^*(Tx)| : x^* \in X^*, x \in X, \|x^*\| = \|x\| = x^*(x) = 1\}$$

and the *numerical index* of X is given by $n(X) := \inf\{v(T) : T \in \mathcal{L}(X), \|T\| = 1\}$.
The problem of computing the numerical index of the L_p-spaces has been latent since the beginning of the theory. It is known that $n(L_p(\mu)) = \inf\{n(l_p^m) : m \in \mathbb{N}\}$ for every measure μ such that $dim(L_p(\mu)) = \infty$. In this talk I will be showing that

$$n(l_p^2) = \sup_{t\in[0,1]} \frac{|t^{p-1} - t|}{1 + t^p} \text{ for } p \in [1.4547, 3.19925].$$

This result is an extension of the result by Javier Merí and Alicia Quero. The computation of two dimensional case is the first step in the computation of $n(l_p)$, but it is reasonable to expect that the sequence $\{n(l_p^m)\}$ is always constant as it happens in the cases $p = 1, 2, \infty$. So computation of $n(l_p^2)$ is most important step and the above result is the best result known so far.