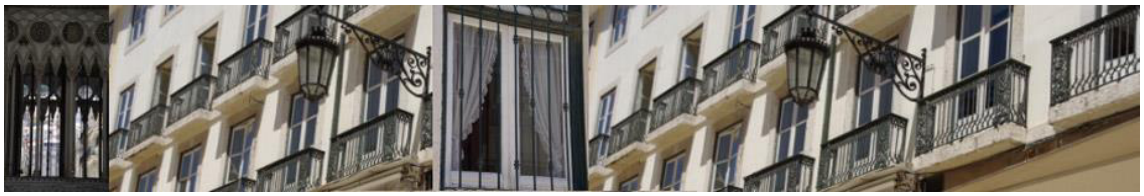
# WOTCA 2021

Workshop on Operator Theory and Complex Analysis

## BOOK OF ABSTRACTS



Department of Mathematics
Instituto Superior Técnico – Universidade de Lisboa, Portugal
June 21-24, 2021

# Contents

## The quantum mechanical setting of a system described by a non-Hermitian Hamiltonian

**Natália Bebiano**[1,2]

*University of Coimbra*[1]
*Research institute/CMUC*[2]

**E-mail address:** *bebiano@mat.uc.pt*

### Abstract

Non-Hermitian Hamiltonians with real spectra appeared in quantum mechanics in the last two decades, originating many difficulties and conflicts with the standard quantum mechanical setting. This event motivated the investigation by mathematicians and physicists of this research area. In this talk it is described how to overcome the main problems.

**Keywords:** Hamiltonian, Non-Hermitian, Eigenvalues.

## Proximinality of subspaces and the quotient lifting property

**Fernanda Botelho**[1]

*Department of Mathematical Sciences*
*The University of Memphis*[1]

**E-mail address:** *mbotelho@memphis.edu*

### Abstract

In this talk I will explain the quotient lifting property (QLP) for pairs of Banach spaces and describe several illustrative examples of such pairs. I will show that proximinality is necessary for the QLP and conditions that imply the QLP will be discussed.

This talk is based on joint work with Monika and Richard Fleming.

**Keywords:** The quotient lifting property; proximinality.

## Farey sequences, continued fractions and BPS black holes

**Gabriel Lopes Cardoso**[1,2]

*Instituto Superior Técnico/Universidade de Lisboa*[1]
*CAMGSD*[2]

**E-mail address:** *gabriel.lopes.cardoso@tecnico.ulisboa.pt*

### Abstract

We use tools from analytic number theory to show how the exact microstate degeneracy of dyonic single-centre $N = 4$ BPS black holes in four space-time dimensions is encoded in the continued fraction of one specific rational number.

**Keywords:** continued fractions, Mock modular forms, BPS black holes.

# The design of operator theory algorithms and the creation of a domain specific language

**Ana C. Conceição**[1,2], **Paula V. Martins**[1,3], **Tiago Côrrea**[1]

*Universidade do Algarve*[1]
*Center for Functional Analysis, Linear Structures and Applications*[2]
*Research Centre for Tourism, Sustainability and Well-being*[3]

**E-mail address:** *aconcei@ualg.pt; pventura@ualg.pt; tiagoc.mail@gmail.com*

## Abstract

In our work we use the computer algebra system *Mathematica* to implement analytical algorithms developed by us and others within the operator theory. The design of our algorithms is focused on the possibility of implementing on a computer all, or a significant part, of the extensive symbolic and numeric calculations present in the analytical algorithms. The methods developed rely on innovative techniques of operator theory and have a potential of extension to more complex and general problems. By implementing these methods on a computer, new tools are created for exploring that same potential, making the results of lengthy and complex calculations available in a simple way to researchers of different areas.

The design of these kind of operator theory algorithms motivate the creation of a domain specific language (DSL) focused on supporting the solutions to problems based on operator theory. In fact, since most operator theory researchers are part of a group of non-programming experts, we decided to create and to provide them with a language that aid the design of these type of algorithms for several classes of functions (in the scalar and matrix cases) and/or special classes of singular integral operators. The idea is to provide a simple and efficient textual language to formalize mathematical models related with the problematic of operator theory. Our DSL present several innovative aspects since it is the first textual language tailored to the computation of singular integrals without implementation details. The possibility to express the algorithm at a higher-level of abstraction is one of the most important features of this domain specific language. The use of the proposed language will allow to implement the models for this class of problems in a user-friendly environment.

**Keywords:** computer algebra system *Mathematica*, domain specific language, operator theory algorithms.

# On the projective space of a Hilbert module

**E. Andruchow, Gustavo Corach, L. Recht**

*Fac. de Ingeniería-Universidad de Buenos Aires*
*Instituto Argentino de Matemática Alberto P. Calderón- CONICET, Argentina*

**E-mail address:** *gcorach@fi.uba.ar*

## Abstract

The projective space of a right Hilbert module X over a C* algebra A is the set of all singly generated submodules [x] which are orthocomplemented with respect to the A-valued inner product. It is the base of a fibration whose total space has a rich differential geometry. We present some results on its geodesics.

# Conditional positive definiteness as a bridge between $k$–hyponormality and $n$–contractivity

**Raúl E. Curto**[1,2]

*University of Iowa*[1]
*Department of Mathematics*[2]

**E-mail address:** *raul-curto@uiowa.edu*

## Abstract

For sequences $\alpha \equiv \{\alpha_n\}_{n=0}^{\infty}$ of positive real numbers, called weights, we study the weighted shift operators $W_\alpha$ having the property of moment infinite divisibility ($\mathcal{MID}$); that is, for any $p > 0$, the Schur power $W_\alpha^p$ is subnormal.

We first prove that $W_\alpha$ is $\mathcal{MID}$ if and only if certain infinite matrices $\log M_\gamma(0)$ and $\log M_\gamma(1)$ are conditionally positive definite (CPD). Here $\gamma$ is the sequence of moments associated with $\alpha$, $M_\gamma(0), M_\gamma(1)$ are the canonical Hankel matrices whose positive semi-definiteness determines the subnormality of $W_\alpha$, and log is calculated entry-wise (i.e., in the sense of Schur or Hadamard).

Next, we use conditional positive definiteness to establish a new bridge between $k$–hyponormality and $n$–contractivity, which sheds significant new light on how the two well known staircases from hyponormality to subnormality interact. As a consequence, we prove that a contractive weighted shift $W_\alpha$ is $\mathcal{MID}$ if and only if for all $p > 0$, $M_\gamma^p(0)$ and $M_\gamma^p(1)$ are CPD.

The talk is based on recent research with Chafiq Benhida (Université de Lille) and George R. Exner (Bucknell University).

**Keywords:** subnormal weighted shift, moment infinitely divisible, conditionally positive definite.

---

# Convolution equations on the Abelian group $\mathcal{A}(-1,1)$ and their applications

**R. Duduchava**

*The University of Georgia*
*A.Razmadze Mathematical Institute, Tbilisi, Georgia*

**E-mail address:** *RolDud@gmail.com*

## Abstract

The interval $\mathcal{J} := [-1,1]$ turnes into the Abelian group $\mathcal{A}(\mathcal{J})$ if endowed with the group operation $x +_{\mathcal{J}} y := (x+y)(1+xy)^{-1}$, $x, y \in \mathcal{J}$. The invariant Haar measure is $d_{\mathcal{J}}x := (1-x^2)^{-1}dx$ and the Fourier transformation is

$$(\mathcal{F}_{\mathcal{J}}v)(\xi) := \int_{-1}^{1} \left(\frac{1-y}{1+y}\right)^{i\xi} \frac{v(y)dy}{1-y^2} = \int_{-1}^{1} \left(\frac{1-y}{1+y}\right)^{i\xi} v(y)d_{\mathcal{J}}y, \qquad \xi \in \mathbb{R}. \tag{1}$$

These tools allow to solve exactly convolution equations on this group

$$c_0 u(x) + \int_{-1}^{1} k\left(\frac{x-y}{1-xy}\right) \frac{v(y)dy}{1-y^2} = h(x), \qquad x \in \mathcal{J}. \tag{2}$$

To the class of equations (2) belongs

$$\sum_{k=0}^{n}\left[a_k(t)\mathfrak{D}^k u(t) - b_k(t)\int_{-1}^{1}\left(\frac{1-\tau^2}{1-t^2}\right)^{d_k}\frac{\mathfrak{D}^k u(\tau)d\tau}{\tau - t}\right] = f(t), \qquad t\in\mathcal{J}, \tag{3}$$

where $d_k \in \mathbb{C}$ are complex numbers, coefficients are sufficiently smooth. Here $\mathfrak{D}u(x) := (1-x^2)\dfrac{d}{dx}u(x)$ is the natural derivative of functions on the group $\mathcal{A}(-1,1)$.

It turned out that to the class of convolution equations (2) belong the following cebrated equations with important applications-Prandtl equation

$$\boldsymbol{P}u(x) = \frac{c_0 u(x)}{1-x^2} + \frac{c_1}{\pi i}\int_{-1}^{1}\frac{u'(y)dy}{y-x} = f(x), \qquad x\in\mathcal{J} \tag{4}$$

Singular Tricomi equation

$$\boldsymbol{T}v(x) = c_0 v(x) + \frac{c_1}{\pi i}\int_{-1}^{1}\frac{v(y)dy}{y-x} + \frac{c_2}{\pi i}\int_{-1}^{1}\frac{v(y)dy}{1-xy} = g(x), \qquad x\in\mathcal{J} \tag{5}$$

and also Lavrentjev-Bitsadze equation. Moreover, Laplace-Beltrami equation on the unit sphere in $\mathbb{S}^2 \subset \mathbb{R}^3$ is also a $\mathcal{J}$-convolution operator with a parameter.

These equations have ample of applications in Mechanics and Mathematical physics and were investigated by many authors. Equations (4) and (5) were investigated by V. E. Petrov in [1, 2] in the general Banach spaceless setting, while in [3] equation (1) was investigated in the Bessel potential space setting $\widetilde{\mathbb{H}}^s(\mathcal{J})$, $-1 \leqslant s \leqslant 1$. These equations were considered as the image of classical Fourier convolutions on $\mathbb{R}$ under the diffeomorphism $x = -\tanh t \,:\, \mathbb{R}\to\mathcal{J}$ mapping the real axes $\mathbb{R}$ to the interval $\mathcal{J}$.

We solve equations (2)-(5) in the full scale of Bessel potential spaces $\mathbb{H}_p^s(\mathcal{J}, d_{\mathcal{J}}x)$ for $-\infty < s < \infty$, $1 < p < \infty$. For an integer $s = m = 1, 2, \ldots$, $\mathbb{H}_p^m(\mathcal{J}, d_{\mathcal{J}}x)$ coincides with the Sobolev space $\mathbb{W}_p^m(\mathcal{J}, d_{\mathcal{J}}x)$ of functions which have weighted $p$-integrable derivatives $\mathfrak{D}^k u \in \mathbb{L}_p(\mathcal{J}, d_{\mathcal{J}}x)$ for $k = 1, 2\ldots, m$.

## References


[1] Petrov V. E.,*J. Math. Sci.* **132** (2006), no. 4, 451 - 481.

[2] Petrov V. E., *Dokl. Math.* **74** (2006), no. 3, 901 - 905.

[3] Suslina T.A., Petrov V.E., https://arxiv.org/abs/2008.06715, 17 pages, 2020.


---

# Finite rank perturbations of normal operators: Spectral subspaces and Borel series


**E. A. Gallardo - Gutiérrez**[1,2] **and F.J. González - Doña**[1,2]

*Universidad Complutense de Madrid*[1]
*Instituto de Ciencias Matemáticas ICMAT*[2]

**E-mail address:** *eva.gallardo@mat.ucm.es; javier.gonzalez@icmat.es*



## Abstract

In this talk, we will address the problem regarding the existence of nontrivial closed invariant subspaces of rank-one (or finite-rank) perturbations of diagonalizable normal operators acting boundedly on a separable, infinite dimensional complex Hilbert space. We will characterize the spectral subspaces associated to closed sets of such operators and prove that a large class of them have non-trivial closed hyperinvariant subspaces (whenever they are not an scalar multiple of the identity), extending previous theorems of Foias, Jung, Ko and Pearcy [2, 3, 4] and of Fang and J. Xia [1]. (Joint work with F. J. González-Doña)



**Keywords:** Rank-one perturbation of normal operators, Spectral subspaces, Borel series.

## References

[1] Q. Fang and J. Xia (2012) Invariant subspaces for certain finite-rank perturba- tions of diagonal operators, *J. Funct. Anal.* 263 no. 5, 1356–1377.

[2] C. Foias, I. B. Jung, E. Ko and C. Pearcy (2007) On rank-one perturbations of normal operators I, *J. Funct. Anal.* 253, no. 2, 628–646.

[3] C. Foias, I. B. Jung, E. Ko and C. Pearcy (2008) On rank-one perturbations of normal operators II, *Indiana Univ. Math. J.* 57 no. 6, 2745–2760.

[4] C. Foias, I. B. Jung, E. Ko and C. Pearcy (2011) Spectral decomposability of rank-one perturbations of normal operators, *J. Math. Anal. Appl.* 375 no. 2, 602–609.


---

# Cesàro-like operators on spaces of analytic functions


**Daniel Girela**

*Universidad de Málaga, Spain*

**E-mail address:** *girela@uma.es*



## Abstract

Let $\mathbb{D}$ be the unit disc in $\mathbb{C}$ and let $\mathcal{H}(\mathbb{D})$ be the space of all analytic functions in $\mathbb{D}$. If $\mu$ is a finite positive Borel measure on the radius $[0,1)$, we define the operator $C_\mu : \mathcal{H}(\mathbb{D}) \to \mathcal{H}(\mathbb{D})$ as follows:

If $f \in \mathcal{H}(\mathbb{D})$, $f(z) = \sum_{n_0}^{\infty} a_n z^n$ $(z \in \mathbb{D})$, we set

$$C_\mu(f)(z) = \sum_{n=0}^{\infty} \mu_n \left( \sum_{k=0}^{n} a_k \right) z^n, \quad z \in \mathbb{D},$$

where, for $n \geq 0$, $\mu_n$ denotes the $n$-th moment of the measure $\mu$, that is, $\mu_n \int_{[0,1)} t^n d\mu(t)$. When $\mu$ is the Lebesgue measure on $[0,1)$, the operator $C_\mu$ reduces to the classical Cesàro operator.

We study the action of the operators $C_\mu$ on distinct spaces of analytic functions in $\mathbb{D}$.

This talk is based on a work in progress in collaboration with Petros Galanopoulos and Noel Merchán.



**Keywords:** Cesáro-like operator, Hardy spaces, $BMOA$, the Bloch space.


---

# Complex analysis solving a control problem


**A. Hartmann**[1,2] **and M.-A. Orsoni**[1,2]

*University of Bordeaux*[1]
*Institut de Mathématiques de Bordeaux*[2]

**E-mail address:** *Andreas.Hartmann@math.u-bordeaux.fr*
*Marcu-Antone.Orsoni@math.u-bordeaux.fr*



## Abstract

Given two spaces of holomorphic functions $X_1$ and $X_2$ defined respectively on domains $\Omega_1$ and $\Omega_2$ of the complex plane with non empty intersection, it is natural to investigate the sum $X_1 + X_2$. This question, also called *separation of singularities problem*, has been

discussed for instance for the Hardy spaces of uniformly bounded functions, $X_k = H^\infty(\Omega_k)$, and for Smirnov spaces (which are companion spaces of Hardy spaces $H^p$). In this talk we will be interested in Bergman spaces $A^p(\Omega_k)$. It is clear that $A^p(\Omega_1)+A^p(\Omega_2) \subset A^p(\Omega_1 \cap \Omega_2)$, and we will discuss situations when this inclusion is an equality. This question is strongly related to control problems. Indeed, in case one wants to control the heat equation on a finite rod, represented by an interval $I$, with ($L^2$-)control at the two endpoints, a seminal work by Fattorini and Russell in the 70's shows that the reachable states extend to holomorphic functions in the square $D$ having $I$ as a diagonal. Recently, a quite important activity took place in order to get a more precise information on these reachable states. Inspired first by work by Dardé-Ervedoza, results by Hartmann-Kellay-Tucsnak and then Orsoni show that a function on $I$ ("a temperature distribution" in the rod) can be reached if it is in the sum of two Bergman spaces defined on angular sectors the intersection of which is the square $D$. In this talk I will show show that in this case the sum of the Bergman spaces is exactly the Bergman space on the square $D$, in other words the reachable space is exactly this Bergman space. We will also consider more general settings for the separation of singularities problem in the Bergman space.

**Keywords:** heat equation, Bergman space, separation of singularities.

## Unbounded Toeplitz operators with rational symbols and their adjoint

**Sanne ter Horst**[1,2]

*North-West University*[1]
*Research Focus Area: Pure and Applied Analytics*[2]

**E-mail address:** *sanne.terhorst@nwu.ac.za*

### Abstract

In this talk we consider an unbounded Toeplitz operator $T_\omega$ on $H^p$ with a rational symbol $\omega$ that has poles on the unit circle. The adjoint operator is determined and it turns out that, for $p = 2$ and with proper symbols, these adjoint operators are a type of unbounded Toeplitz operator studied by Sarason. Furthermore, again for $p = 2$, it is studied when $T_\omega^*$ is symmetric and when $T_\omega^*$ has a selfadjoint extension. The talk is joint work with Gilbert Groenewald, Jacob Jaftha and André Ran.

**Keywords:** Unbounded Toeplitz operators, adjoint, symmetric operators, selfadjoint extensions.

## Generalizations of Wigner's unitary–antiunitary theorem

**D. Ilišević**[1], **A. Turnšek**[2,3] **and M. Omladič**[3]

*University of Zagreb, Croatia*[1]
*University of Ljubljana, Slovenia*[2]
*Institute of Mathematics, Physics and Mechanics, Ljubljana, Slovenia*[3]

**E-mail address:** *ilisevic@math.hr; aleksej.turnsek@fpp.uni-lj.si; matjaz@omladic.net*

### Abstract

Let $(H, (\cdot,\cdot))$ and $(K, (\cdot,\cdot))$ be real or complex inner product spaces. The celebrated Wigner's unitary–antiunitary theorem states that a mapping $f\colon H \to K$ satisfies

$$|(f(x), f(y))| = |(x, y)|, \quad x, y \in H,$$

if and only if there is a linear or an anti-linear isometry $U\colon H \to K$ such that

$$f(x) = \sigma(x)Ux, \quad x \in H,$$

where a so-called phase function $\sigma$ takes values in modulus one scalars; in other words, $f$ is phase equivalent to a linear or an anti-linear isometry. In this talk several generalizations of this theorem to normed spaces will be presented.

**Keywords:** Wigner's theorem, normed space, isometry.

---

# Equality of the norm, the essential norm, and the measure of noncompactness of two interesting operators

**Alexei Karlovich**[1,2] **and Eugene Shargorodsky**[3]

*Universidade NOVA de Lisboa*[1]
*Centro de MatemÃ¡tica e Aplicações*[2]
*King's College London*[3]

**E-mail address:** *oyk@fct.unl.pt*

## Abstract

Let $X$ be a rearrangement-invariant Banach function space on the unit circle $\mathbb{T}$ and let $H[X]$ be the abstract Hardy space built upon $X$. We prove that if the Cauchy singular integral operator $(Hf)(t) = \frac{1}{\pi i}\int_{\mathbb{T}} \frac{f(\tau)}{\tau - t}\,d\tau$ is bounded on the space $X$, then the norm, the essential norm, and the measure of non-compactness of the operator $aI + bH$ with $a, b \in \mathbb{C}$, acting on the space $X$, coincide. We also show that the backward shift operator $(Sf)(t) = (f(t) - \widehat{f}(0))/t$ is bounded on the abstract Hardy space $H[X]$ and its norm, its essential norm, and its measure of non-compactness are all the same. Our results extend those by Krupnik and Polonskii (1975) for the operator $aI + bH$ and by Shargorodsky (2021) for the operator $S$.

**Keywords:** Abstract Hardy space, backward shift operator, Cauchy singular integral operator, norm essential norm, measure of noncompactness.

---

# The characterization of dual truncated Toeplitz operators

**Kamila Kliś-Garlicka**[1]

*University of Agriculture in Krakow, Poland*[1]

**E-mail address:** *rmklis@cyfronet.pl*

## Abstract

Consider a nonconstant inner function $\theta$. A dual truncated Toeplitz operator is the restriction of a multiplication operator on $L^2$ on the unit circle to $K_\theta^\perp$ the orthogonal complement of the model space $K_\theta$. The necessary and sufficient conditions for any operator on $K_\theta^\perp$ to be a dual truncated Toeplitz is given. The asymmetric case, i.e., operators between orthogonal complements of different model spaces, is also considered.

Joint work with C. Câmara, B. Łanucha and M. Ptak.

---

# Spectral analysis of Dirac and Pauli operators

**David Krejčiřík**

*Czech Technical University in Prague*

**E-mail address:** *David.Krejcirik@fjfi.cvut.cz*

## Abstract

By developing the method of multipliers, we establish sufficient conditions on the magnetic field and the complex, matrix-valued electric potential, which guarantee that the corresponding system of Schrödinger operators has no point spectrum. In particular, this allows us to prove analogous results for Pauli operators under the same electromagnetic conditions and, in turn, as a consequence of the supersymmetric structure, also for magnetic Dirac operators. This is joint work with Lucrezia Cossetti, Luca Fanelli and Luis Vega.

**Keywords:** method of multipliers, relativistic Hamiltonians, absence of eigenvalues, non-self-adjointness.

## References

[1] L. Cossetti, L. Fanelli and D. Krejčiřík (2020) Absence of eigenvalues of Dirac and Pauli Hamiltonians via the method of multipliers, *Comm. Math. Phys.* **379**, 633–691.

[2] L. Fanelli, D. Krejčiřík and L. Vega (2019) Location of eigenvalues of three-dimensional non-self-adjoint Dirac operators, *Lett. Math. Phys.* **109**, 1473–1485.

[3] L. Fanelli, D. Krejčiřík and L. Vega (2018) Absence of eigenvalues of two-dimensional magnetic Schrödinger operators, *J. Funct. Anal.* **275**, 2453–2472.

---

# $k^{th}$–order slant Toeplitz operators and their compressions to model spaces

**Bartosz Łanucha**[1] **and Małgorzata Michalska**[1]

*Maria Curie-Skłodowska University, Lublin, Poland*[1]

**E-mail address:** *bartosz.lanucha@poczta.umcs.lublin.pl*

## Abstract

A $k^{th}$–order ($k \geq 1$) slant Toeplitz operator $U_\varphi : L^2 \to L^2$ is defined as the operator represented by the matrix (with respect to the standard basis) $(a_{ki-j})_{i,j}$, where $(a_n)$ are the Fourier coefficients of $\varphi \in L^\infty$. If $k = 2$, then $U_\varphi$ is called a slant Toeplitz operator. Note that for $k = 1$ the operator $U_\varphi$ is the multiplication operator $M_\varphi$: $f \mapsto \varphi f$.

Here we consider compressions of $k^{th}$–order slant Toeplitz operators to model spaces, that is, the backward shift invariant subspaces of the classical Hardy space $H^2$. We investigate when these compressions are equal to the zero operator and present characterizations using compressed shifts and finite rank operators of special kind. We also investigate some commuting relations for compressions of $k^{th}$–order slant Toeplitz operators.

**Keywords:** model space, compressed shift, Toeplitz operator, slant Toeplitz operator, generalized slant Toeplitz operator, truncated Toeplitz operator.

# Toeplitz operators on the Newton spaces

**Eungil Ko[1], Ji Eun Lee[2], and Jongrak Lee[3]**

*Ewha Womans University, Seoul, Korea*[1]
*Sejong University, Seoul, Korea*[2]
*Jeju National University, Jeju, Korea*[3]

**E-mail address:** *jieunlee7@sejong.ac.kr: jieun7@ewhain.net*

## Abstract

In this paper, we study the Newton space $N^2(\mathbb{P})$ which has the Newton polynomials as an orthonormal basis. We first investigate some relations between the orthonormal basis $\{z^n\}$ of the Hardy space $H^2(\mathbb{D})$ and the orthonormal basis $\{N_n\}$ of the Newton space $N^2(\mathbb{P})$. Moreover, we consider the properties of Toeplitz operators on Newton space $N^2(\mathbb{P})$.

**Keywords:** Newton space, Newton polynomials, Toeplitz operator.

# On the kernel of a singular integral operator with a non-Carleman shift

**Rui Marreiros[1,2]**

*University of Algarve*[1]
*Center for Functional Analysis, Linear Structures and Applications - CEAFEL; IST*[2]

**E-mail address:** *rmarrei@ualg.pt*

## Abstract

We consider the singular integral operator with a non-Carleman shift $T = I - cUP_+$: $L^n_p(\mathrm{T}) \to L^n_p(\mathrm{T})$, $p \in (1, \infty)$, where $P_+$ is the Cauchy projector, $U$ is an isometric shift operator and $c(t)$ is a continuous matrix function on the unit circle T. We obtain some estimates for the dimension of the kernel of the operator $T$.

**Keywords:** Singular integral operators, shift operators, kernel dimension.

# A new class of infinite matrices

**Javad Mashreghi[1,2] Ludovick Bouthat[1]**

*Laval University, Quebec*[1]
*CRM, Montreal*[2]

**E-mail address:** *javad.mashreghi@mat.ulaval.ca*

## Abstract

In studying the Hadamard multipliers of weighted Dirichlet spaces, we face with infinite L-shaped matrices which act as bounded operators on the Hilbert space $ell^2$. For such matrices, we usually have an estimation of the norm. This class is no exception and we provide some bounds for the norm. However, our techniques are optimal since in some special cases we succeed to precisely evaluate the norm.

**Keywords:** Hadamard multipliers, L-matrix, norm.

# On a Fréchet functional equation over nonarchimedean normed spaces

**Sérgio Mendes**[1,2] **and Gastão Bettencourt**[3,4]

*ISCTE - University Institute of Lisbon*[1]
*Centro de Matemática e Aplicações - UBI*[2]
*Universidade da Beira Interior*[3]
*Centro de Matemática e Aplicações - UBI*[4]

**E-mail address:** *smnm@iscte-iul.pt; gastao@ubi.pt*

## Abstract

The first norm characterization of inner product spaces was given by Fréchet in 1935. In 1936 Jordan and von Neumann proved that a normed space $X$ is an inner product space if and only if the parallelogram law holds in $X$. Since then many other characterizations have been proved.

In this talk we study Hyers-Ulam stability for a Fréchet functional equation on nonarchimedean normed spaces. If time permits, a brief digression on nonarchimedean Hilbert spaces will also be presented.

**Keywords:** Hyers-Ulam stability, nonarchimedean Fréchet functional equation, length function.

# On some properties of weighted algebras of vector-valued continuous functions

**Lourdes Palacios**[1,2]

*Universidad Autónoma Metropolitana*[1]
*Departamento de Matemáticas/Unidad Iztapalapa*[2]

**E-mail address:** *pafa@xanum.uam.mx*

## Abstract

Let $X$ be a completely regular Hausdorff space and $V$ a Nachbin family on X. For $A$ a locally convex algebra, let $CV_{(0)}(X, A)$ be the algebra of all weighted vector-valued continuous functions with the topology given by the uniform seminorms induced by $V$. In this talk we present some examples of such spaces and some properties of $A$ that are inherited by $CV_{(0)}(X, A)$. These properties are related to the $m$-convex property, the $Q$ property and the uniformly $A$-convex property.

**Keywords:** Vector valued continuous functions spaces, $m$-convex algebra, uniformly A-convex algebra.

# A decomposition of function spaces, with applications

**Jonathan R. Partington**[1]

[1] *School of Mathematics, University of Leeds, Leeds LS2 9JT, UK*

**E-mail address:** *J.R.Partington@leeds.ac.uk*

## Abstract

Let $B$ be a finite Blaschke product and $K_B = H^2 \ominus BH^2$ the associated finite-dimensional model space. For a scale of function spaces, including Hardy, Bergman, and Dirichlet, we have decompositions of the form

$$X = K_B \oplus BK_B \oplus B^2K_B \oplus \dots$$

with appropriate norm estimates. The applications include: weighted composition operators [1], commutants of multipliers [2], and wandering subspaces [3].

This is joint work with Eva Gallardo-Gutiérrez, with additional material from Isabelle Chalendar and Daniel Seco.

**Keywords:** Blaschke product, model space, multiplier, commutant, weighted composition operator.

## References

[1] I. Chalendar, E.A. Gallardo-Gutiérrez and J.R. Partington (2015) Weighted composition operators on the Dirichlet space: boundedness and spectral properties, *Math. Annalen* **363**, 1265–1279.

[2] E.A. Gallardo-Gutiérrez and J.R. Partington (2021) Multiplication by a finite Blaschke product on weighted Bergman spaces: commutant and reducing subspaces, *preprint*, https://arxiv.org/abs/2105.07760.

[3] E.A. Gallardo-Gutiérrez, J.R. Partington and D. Seco (2020) On the wandering property in Dirichlet spaces, *Integral Equations and Operator Theory* **92**, paper no. 16.


---

# Characterizations of spectral measures on the real line


**Paweł Pietrzycki**[1] **and Jan Stochel**[1]

*Jagiellonian University, Kraków, Poland*[1]

**E-mail address:** *pawel.pietrzycki@im.uj.edu.pl; jan.stochel@im.uj.edu.pl*



## Abstract

In [1], Kiukas, Lahti and Ylinen asked the following general question. *When is a positive operator measure projection valued?* The following version of this question formulated in terms of operator moments was posed in [2]. *Let $T$ be a selfadjoint operator and $F$ be a Borel semispectral measure on the real line with compact support. For which positive integers $p < q$ do the equalities $T^k = \int_R x^k F(dx)$, $k = p, q$, imply that $F$ is a spectral measure?*

We will discuss recent results concerning these problems obtained in [2, 3]. In particular, we will present a complete solution to the second problem. Namely, the answer is affirmative if $p$ is odd and $q$ is even, and negative otherwise. The case $(p, q) = (1, 2)$ closely related to intrinsic noise operator was solved by several authors including Kruszyński and de Muynck as well as Kiukas, Lahti and Ylinen.



**Keywords:** Semispectral measure, spectral measure, operator moment.



## References

[1] J. Kiukas, P. Lahti, K. Ylinen, Phase space quantisation and the operator moment problem, *J. Math. Phys.* **47** (2006), 072104.

[2] P. Pietrzycki, J. Stochel, Subnormal $n$th roots of quasinormal operators are quasinormal, *J. Funct. Anal.* **280** (2021), 109001.

[3] P. Pietrzycki, J. Stochel, Two-moment characterization of spectral measures on the real line, preprint.


---

# Shift invariance and reflexivity of compressions of multiplication operators


**C. Câmara**[1,2]**, K. Kliś-Garlicka**[3]**, B. Łanucha**[4] **and M. Ptak**[3]

*Instituto Superior Tecnico, Universidade de Lisboa, Portugal,*[1]
*Center for Mathematical Analysis, Geometry and Dynamical Systems*[2]
*University of Agriculture in Krakow, Poland,*[3]
*Maria Curie-Skłodowska University, Lublin, Poland.*[4]

**E-mail address:** *ccamara@math.ist.utl.pt; rmklis@cyf-kr.edu.pl; bartosz.lanucha@poczta.umcs.lublin.pl; rmptak@cyf-kr.edu.pl*

## Abstract

The classical Toeplitz operators have the specific behaviour with respect to the unilateral shift. This behaviour allows to characterize them and as a consequence we obtain some reflexivity results. Truncated Toeplitz operators in the model spaces have, not exactly the same but, similar behaviour. A property of being shift invariant and being reflexive or transitive in the case of a space of (asymmetric) truncated Toeplitz operators, and a space of (asymmetric) dual truncated operators , Toeplitz operators on the orthogonal complement to model spaces, will be presented. Most of the results obtained are new even for the symmetric case.

**Keywords:** asymmetric truncated Toeplitz operator, dual truncated Toeplitz operator, reflexivity, transitivity, $k$–reflexivity.

---

# Eigenvalues of rank one perturbations of matrices: global properties

**André Ran**[1,2] **and MichałWojtylak**[3]

*Vrije Universiteit Amsterdam*[1]
*North West University, South Africa*[2]
*Uniwersytet Jagielloński*[3]

**E-mail address:** *a.c.m.ran@vu.nl; michal.wojtylak@gmail.com*

## Abstract

Given a complex $n \times n$ matrix $A$ and complex vectors $u$ and $v$ in $\mathbb{C}^n$, we consider in this talk general properties of eigenvalues of $A + \tau uv^*$ as functions of either $\tau \in \mathbb{C}$ or $\tau \in \mathbb{R}$, or $\tau = e^{i\theta}$ on the unit circle. In particular, the limits of eigenvalues with $\tau \to \infty$ are discussed in detail, and the behaviour of the eigenvalues as functions of $\tau$ for large values of $|\tau|$ is considered. It turns out that a very precise description can be given, which is both qualitative and quantitative in nature.

**Keywords:** Eigenvalue perturbation theory

---

# Square roots of classical operators

**William T. Ross**[1]

*University of Richmond*[1]

**E-mail address:** *wross@richmond.edu*

## Abstract

A square root of a bounded Hilbert space operator $A$ is a bounded operator $B$ such that $B^2 = A$. Not every operator has a square root and some operators have many square roots. In joint with with Marek Ptak and Javad Mashreghi, we explore the square roots of some classical operators and discuss which ones have square roots. For the ones which have square roots, we describe them all. The operators explored in this talk are the classical shift $S$, its square $S^2$, the Volterra operator, the Cesáro operator, and the Toeplitz operator $T_{\cos\theta}$.

**Keywords:** Hilbert space operators, Hardy space, shift operators, Volterra operator, Cesàro operator, Toeplitz operators.

---

# On the essential norms of Toeplitz operators

**Eugene Shargorodsky**[1,2] **and Alexei Karlovich**[3]

*King's College London*[1]
*Technische Universität Dresden*[2]
*Universidade Nova de Lisboa*[3]

**E-mail address:** *eugene.shargorodsky@kcl.ac.uk; oyk@fct.unl.pt*

## Abstract

It is well known that the essential norm of a Toeplitz operator on the Hardy space $H^p(\mathbb{T})$, $1 < p < \infty$ is greater than or equal to the $L^\infty$ norm of its symbol. In 1988, A. Böttcher, N. Krupnik, and B. Silbermann posed a question on whether or not the equality holds in the case of continuous symbols. This question has been recently answered in the negative (for $p \neq 2$). On the other hand, it was shown that the essential norm of a Toeplitz operator $T(a)$ with a continuous symbol $a$ is less than or equal to $2^{\left|1-\frac{2}{p}\right|}\|a\|_{L^\infty}$. We extend these results to weighted Hardy spaces and their generalisations. We also discuss some open questions related to the above estimates.

**Keywords:** measure of noncompactness, bounded compact approximation property, Banach function spaces.

---

# On the Stampfli point of some operators and matrices

**Ilya M. Spitkovsky**[1,2] **and Thanin Quartz**[3,4]

*New York University Abu Dhabi (NYUAD)*[1]
*Division of Science and Mathematics*[2]
*University of Waterloo*[3]
*Department of Applied Mathematics*[4]

**E-mail address:** *ims2@nyu.edu; tquartz@uwaterloo.ca*

## Abstract

The center of mass of an operator $A$ (denoted $\mathrm{St}(A)$, and called in this presentation the *Stampfli point* of $A$) was introduced by Stampfli in his Pacific J. Math (1970) paper as the unique $\lambda \in \mathbb{C}$ delivering the minimum value of $||A - \lambda I||$. We derive some results concerning the location of $\mathrm{St}(A)$ for several classes of operators, including 2-by-2 block operator matrices with scalar diagonal blocks and 3-by-3 matrices with repeated eigenvalues. We also show that for almost normal $A$ its Stampfli point lies in the convex hull of the spectrum, which is not the case in general. Some relations between the property $\mathrm{St}(A) = 0$ and Roberts orthogonality of $A$ to the identity operator are established.

**Keywords:** center of mass, almost normal operator, Stampfli point.

---

# Spectral bounds for 1D discrete Schrödinger operators with complex potentials

**František Štampach**[1]

*Faculty of Nuclear Science and Physical Engineering,*
*Czech Technical University in Prague*[1]

**E-mail address:** *stampfra@fjfi.cvut.cz*

## Abstract

First, we discuss optimal spectral enclosures for discrete Laplacians on $\mathbb{Z}$ and $\mathbb{N}$ with the Dirichlet boundary condition perturbed by complex $\ell^1$-potentials. Second, we present related results on a spectral stability of discrete Schrödinger operators on $\mathbb{N}$ with small complex potentials.

The talk is based on joint projects with O. O. Ibrogimov, D. Krejčiřík, and A. Laptev.

**Keywords:** discrete Schrödinger operator, spectral enclosure, spectral stability.

---

# An attempt at defining (Toeplitz and) Hankel operators in unbounded domains

**Franciszek Hugon Szafraniec**

*Instytut Matematyki, Uniwersytet Jagielloński, Kraków, Poland*

**E-mail address:** *umszafra@cyf-kr.edu.pl*

## Abstract

Taking advantage of the very general idea of defining Toeplitz and Hankel operators I would like to foster developing main ingredients of those in "unbounded domains"; the latter is a rather rough approximate description of my principal goal though as an example it serves well.

**Keywords:** Toeplitz operator, Hankel operator, general approach.

---

# Reducing subspaces of certain truncated Toeplitz operators

**Dan Timotin**[1]

*Simion Stoilow Institute of Mathematics of the Romanian Academy*[1]

**E-mail address:** *dtimotin@yahoo.com*

## Abstract

The Sz.-Nagy–Foias theory of contractions is a powerful tool for the investigation of Hilbert space operators. We apply it in order to discuss reducibility of completely nonunitary contractions. In particular, we present some recent results concerning reducibility of truncated Toeplitz operators.

This is based on a joint work with Chafiq Benhida and Emmanuel Fricain.

**Keywords:** contractions on Hilbert space, reducibility, truncated Toeplitz operators.

---

# On the asymptotics of block Toeplitz determinants

**Jani Virtanen**[1]

*University of Reading, United Kingdom*[1]

**E-mail address:** *j.a.virtanen@reading.ac.uk*

## Abstract

In this talk, I discuss the determinants of finite Toeplitz matrices with entries $f_{j,k} = f_{j-k}$, where $f_j$ are the Fourier coefficients of the symbol $f$ defined on the unit circle, as the size of the matrices tends to infinity. Their asymptotic behavior is well known for smooth symbols and is of the form $G^n E$, where $n$ is the matrix size, and the constants $G$ and $E$ can be explicitly described in the scalar case. In the block case (when the entries are matrices of fixed size), Harold Widom proved an analogous result and described the constant $E$ as an infinite determinant (known as Widom's constant). The talk discusses computation of Widom's constant and the block case when the symbol is piecewise continuous. Joint work with Estelle Basor and Torsten Ehrhardt.

# Sturm-Liouville problems with eigenparameter dependent transmission conditions

**B.A. Watson**[1], **C. Bartels**[1] **and S. Currie**[1]

*University of the Witwatersrand, South Africa*[1]

**E-mail address:** *b.alastair.watson@gmail.com*

## Abstract

We consider two Sturm-Liouville equations on finite intervals which interact via a transmission matrix. The coefficients of the transmission matrix are generalized Nevanlinna functions of the eigenparameter. This can be posed as an operator eigenvalue problem in $L^2 \bigoplus C^n$, see [1]. We consider factorization of this operator to yield a Darboux-Crum type almost iso-spectral transformation for this problem.

**Keywords:** Sturm-Liouville, Transmission conditions, Darboux-Crum transformation.

## References

[1] C. Bartels, S. Currie, M. Nowaczyk and B.A. Watson (2018) Sturm-Liouville problems with transfer condition Herglotz dependent on the eigenparameter – Hilbert space formulation, *Integral equations and operator theory*, **90**, 1–20.

# Deformed numerical range, dilations, spectral constants

**M. Wojtylak**[1] **P. Pagacz**[1], **P. Pietrzycki**[1]

*Jagiellonian University*
*Kraków*

**E-mail address:** *michal.wojtylak@uj.edu.pl*

## Abstract

For a matrix $T$ we introduce a a deformed numerical range $W_r(T)$, where $r$ is a parameter between 0 and 2. It is a convex set, containing the spectrum, depending continuously on $r$. Other properties of $W_r(T)$ will be shown as well. We will review the dilation theory and discuss the universal spectral constants for these sets, i.e., constants $C_r$ such that

$$\|p(T)\| \leq C_r \sup_{z \in W_r(T)} |p(z)|$$

for any polynomial $p(z)$. We will also show the relation of $W_r(T)$ to other variations on the numerical range, in particular to the normalised numerical range.

**Keywords:** numerical range, functional calculus, spectral constant

# Compact Operators and Projective Spectrum

**Rongwei Yang**

*the State University of New York at Albany*

**E-mail address:** *ryang@albany.edu*

## Abstract

Finitely generated structures are important subjects of study in various mathematical disciplines. Examples include finitely generated groups, Lie algebras and $C^*$-algebras, etc. It is thus a fundamental question whether there exists a universal mechanism in the study of these vastly different entities. In 2009, the notion of projective spectrum for several elements $A_1, ..., A_n$ in a unital Banach algebra $\mathcal{B}$ was defined through the multiparameter pencil $A(z) = z_1A_1 + \cdots + z_nA_n, z \in \mathbb{C}^n$. This conspicuously simple definition turned out to have a surprisingly rich content. This talk focuses on an application of projective spectrum to the study of compact operators, and it will reveal a natural link with complex analysis and algebraic geometry.

This is a joint work with Wei He at Southeastern University, China.

**Keywords:** compact operator, projective spectrum, thin set, kernel bundle, Chern class.